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B.M.S COLLEGE FOR WOMEN
BENGALURU – 560004

I SEMESTER END EXAMINATION – JAN/FEB-2024

B.Sc – MATHEMATICS

ALGEBRA-I AND CALCULUS-I
(NEP Scheme 2021-22 onwards F+R)

Course Code: MAT1DSC01 **QP Code: 1015**
Duration: 2 ½ Hours **Max marks: 60**

Instructions: Answer all the sections.

SECTION-A

I. Answer any SIX questions. Each question carries TWO marks (6 × 2 = 12)

1. Define Symmetric Matrix. Give an Example.
2. Find the value *k* in order that the matrix A = $\begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of Rank 2
3. Find the n^{th} derivative of $log_e(2x + 3)$
4. Discuss the Differentiability of $f(x) = \begin{cases} x sin\left(\frac{1}{x}\right), & x \neq 0 \\ x, & x = 0 \end{cases}$ at x=0
5. Verify Lagrange's Mean Value Theorem for the function $f(x) = (x - 1)(x - 2)$ in $[0, 4]$
6. Evaluate: $\lim_{x \to 0} \frac{\log(sinx)}{cotx}$ using L-Hospital's Rule
7. If $u = \frac{x}{y}$ then Prove That $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
8. Find the Total Derivative du/dt, when $u = x^2 + y^2$, where $x = e^t$, $y = sint$

SECTION-B

II. Answer any TWO questions. Each question carries SIX marks (2×6 = 12)

1. Find the Rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ by reducing it to row reduced echelon form

2. Show that the following equations are consistent and solve them $x + y + z = 6$,

$x + 2y + 3z = 14$ and $x + 4y + 7z = 30$

3. a) Find Eigen Values of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

SECTION-C

III. Answer any Six Questions. Each question carries SIX marks **(6 × 6 = 36)**

1. a) Discuss the Continuity of $f(x) = \frac{1}{1+e^{-1/x}}$, if $x \neq 0$ and $f(0)=0$ at $x=0$

b) Find the n^{th} derivative of $\frac{1}{(x+3)(2x+1)}$

2.If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$ show that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$

3. Prove that the function which is a continuous in a closed interval is bounded

4. State and prove Cauchy Mean Value Theorem

5. a) Expand using Maclaurin's Series $f(x) = log_e(1 + sinx)$

b) Evaluate : $\lim_{x \to 0} \left(\frac{1}{x}\right)^{2tanx}$ using L-Hospital's Rule

6. a) If $u = (x - y)^n + (y - z)^n + (z - x)^n$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

b) If $u = x^2$, $v = y^2$, then find $J = \frac{\partial(u,v)}{\partial(x,y)}$ and $J' = \frac{\partial(x,y)}{\partial(u,v)}$

7. If $u = sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu$

ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u$

8. Find the extreme values of the function : $f(x, y) = x^3 + y^3 - 3x - 12y + 2$.

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